I ELEMENTS OF A DIGITAL ARCHITECTURE

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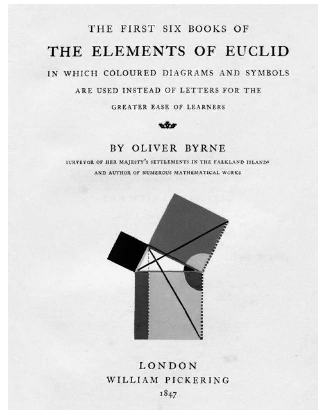
People are both fascinated by and afraid of computers. This text gives you an idea of what the coding of computers is about. Computers are not machines. And because we left the era of machines, architecture no longer is about a geometry of lines, such as it was introduced by Alberti or Palladio in the Renaissance. The elements of today's architecture are of an algebraic nature: they are "whatever can be the case." This text is written as an epos. Its long and adventurous journey is set up by the Timaeus and visits Pythagoras, Ptolemy, Alberti, Lagrange, and finally Markov. By following this journey you take part in the creation of a new geometry of something we might call Digital Man. This man is symmetrical to Renaissance Man, who discovered the modern world and who became so natural to us over the last 500 years. Digital Man opens up a new plateau, which has been fascinating and frightening to all of us since the end of the nineteenth century. You will find an instrument to create your identity within a digital architecture. It incorporates the Euclidean geometry cultivating space, as well as the Cartesian space cultivating time, and by doing exactly that it enables you to move in between times to make your own architecture. This is what the elements of digital architecture are about. This is what all masterful architecture of the last 100 years is about.

What is information? What does "coding" mean? These are questions we, as architects, want to ask ourselves.[1]
For Norbert Wiener, information is neither matter nor energy, and therefore not in space or time. So what is it? We don't want to formulate this question as a problem that we can get to the bottom of and find a solution for. Rather, we see in it a challenge and we meet this challenge with a hypothesis that at first glance may seem somewhat baffling:

Coding is a new form of geometry.



And, as with any new geometry to date—the geometry of Euclid and the geometry of Descartes—this new geometry unlocks a new world.

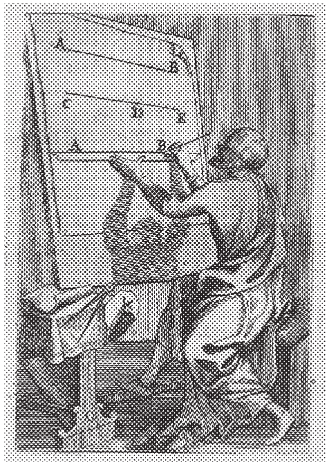
Once we look closely, we are surprised to realize that only analytical geometry is in fact drawn, whereas Euclidean geometry is described by text. The illustrations of Euclidean geometry that we are so familiar with today are in reality a nineteenth-century translation into the representative world of a then current analytical geometry for didactic purposes.

1 *This text gives very few references. If you are interested in more details, you easily can take the given names, concepts, diagrams or images to get references and further readings in the Internet.*

Yet even analytical geometry does not only operate with lines, but primarily with numbers. Where, for example, is the point of intersection of two straight lines through the coordinates:

```
L1((1,-1/2), (2,0))
L2((1,-8), (2,-10))
```

This can be solved as a drawing:



Or it can be solved using this well-known proportional arithmetic:

```
a = dy/dx
a = (y2-y1)/(x2-x1)
a1 = (0+1/2)/(2-1) = 1/2

b = y-a*x
b1 = 0-0.5*2 = -1

y = a*x+b
L1(x) = 1/2*x+-1

a2 = (-10+8)/(2-1) = -2
b2 = -10-(-2)2 = -6

L2(x) = -2x-6

L1(x) = L2(x)
1/2x-1 = -2x-6
2.5*x-1 = -6
```



```
2.5*x = -5
x = -2
y = -2x-6
y = -2*-2-6
y = 4-6
y = -2
```

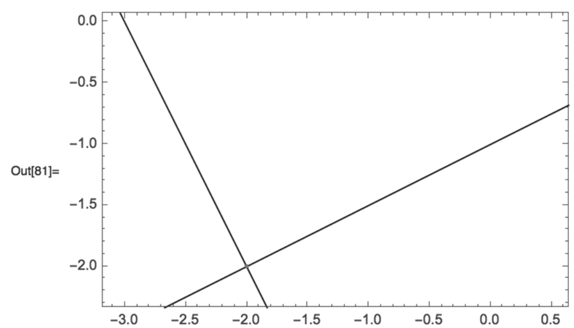
Point of Intersection S(-2,-2)

This procedure is cumbersome, especially when used on complex geometric queries, and indeed, since the advent of computing twenty to thirty years ago, hardly anyone who has learned this at school still actually applies it. Today, this type of query is coded. And the code no longer uses *arithmetics* to measure *geometric elements,* but instead uses *symbols* to operate *with algebraic elements.* Such a code might look something like this:

```
In[73]:=R1 = Infinite Line [{{1, -.5}, {2, 0}}];
        R2 = Infinite Line [{{1, -8}, {2, -10}}];
        Solve[{x, y} ∈ R1 && {x, y} ∈ R2, {x, y}]
In[73]:={{x → -2., y → -2.}}
```

So we now only formulate the parameters of the query; the pathway toward a solution, which in the arithmetic procedure was still of some interest to us, has become generic. And in a similar vein, we now generate the familiar graphic representation:

```
In[73]:=Graphics[{
        {Blue, R1, R2},
        {Red, Point [{x, y}]/.%}]
        Frame → True]
```



We therefore want to distance ourselves from the idea that there is only one fundamental geometry, and that geometry has anything to do with the drawing of lines and circles.

Geometry is the rationalization of thought patterns amid known elements.

Thus we also distance ourselves from the idea of an inflationary number of different geometries, as they are today being delineated: projective, affine, convergent, Euclidean, Non-Euclidean … We would regard none of these as geometries, because they all have come about, just as the didactic illustrations of Euclidean geometry mentioned above, as a result of an "algebraification" of mathematics during the nineteenth century and are not originally geometries. Rather they are—as we would say today—renderings of algebraic expressions into visual-spatial dimensions. And so this plethora of geometries has its origin in algebraic, not geometric, thinking. These are therefore not geometries. They only look like them. At first glance.

It is a different story with digital code. Here, as we have seen, algebraic expressions are being signed, as we would call it. These signatures, not the numbers, are the elements of the code. So if geometry is the rationalization of thought patterns amid known elements, then code is the rationalization of thought patterns amid signatures, the elements of symbolic algebra.

Code is a new geometry. New in the sense that with these *signatures* we align ourselves with *numerals*, which may be regarded as the elements of analytical geometry and *characters*, which may be regarded as the elements of Euclidean geometry.

To the elements of a new geometry correspond new notations: Euclidean geometry develops in tandem with the development of *phonetic* notation; analytical geometry with a mobile, *mechanical* notation, that is, the printing press. Coding develops in tandem with an *operational* notation, that is, computing.

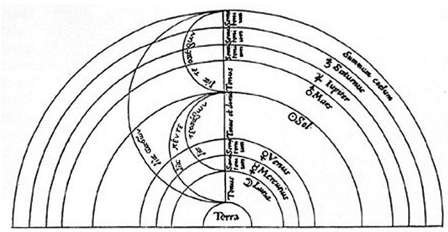
And: a new geometry always unlocks a new world: during the Antique, characterizing things through phonetics unlocks *space*. During modernity, numeration of space through movement unlocks time. And today, we suggest that the signing of *time* through operations unlocks *values*.

EUCLIDEAN GEOMETRY	ANALYTICAL GEOMETRY	CODE AS GEOMETRY
characters	ciphers	signatures
phonetic writing	functional printing	operational coding
space	time	value

The Form and Method of this text are unusual. It is not analytically reflective. Rather, the text posits a symmetrical body of thinking, which, in keeping with group theory in mathematics, utilizes the concepts of associativity, neutrality, and inversion. It follows the hypothesis that, in the tradition of Galois, groups atomize time by means of algebra. Thus we build symmetries to the methodology of Descartes who of a fashion in this way atomized space by means of algebra and captured time by means of geometry, just as Democritus atomized things by means of algebra and by means of geometry captured space. The text then is a symmetrical constellation outside of any time and thus in itself shows the form of a digital architecture.

Certainly, these symmetries may appear far-fetched, and also perhaps somewhat arbitrary. But in the course of this text, akin to a game of sudoku, the symmetries will stabilize without making it necessary to specify the concepts employed. And in this, the ability to keep the concepts alive while still being able to operate with them, lies the particular strength of our new geometry.

So with this text, we want to arrange symmetries in a thought construct and compose a fugue of operational thinking.



From Stanley's *The History of Philosophy.*

THE INTERVALS AND HARMONIES OF THE SPHERES.

In the Pythagorean concept of the music of the spheres, *the interval between the earth and the sphere of the fixed stars was considered to be a diapason—the most perfect harmonic interval. The following arrangement is most generally accepted for the musical intervals of the planets between the earth and the sphere of the fixed stars: From the sphere of the earth to the sphere of the moon, one tone; from the sphere of the moon to that of Mercury, one-half tone; from Mercury to Venus, one-half tone; from Venus to the sun, one and one-half tones; from the sun to Mars, one tone; from Mars to Jupiter, one-half tone; from Jupiter to Saturn, one-half tone; from Saturn to the fixed stars, one-half tone. The sum of these intervals equals the six whole tones of the octave.*

There are very few texts with a similar importance to Western thinking as Plato's *Timaeus*. This is the passage where the demiurge creates the world:

PLATO, *TIMAEUS*, 35 A,
TRANSLATED BY BENJAMIN JOWETT

He took the three elements of the same, the other, and the essence, and mingled them into one form, compressing by force the reluctant and unsociable nature of the other into the same. When he had mingled them with the essence and out of three made one, he again divided this whole into as many portions as was fitting, each portion being a compound of the same, the other, and the essence. And he proceeded to divide after this manner: First of all, he took away one part of the whole [1], and then he separated a second part which was double the first [2], and then he took away a third part which was half as much again as the second and three times as much as the first [3], and then he took a fourth part which was twice as much as the second [4], and a fifth part which was three times the third [9], and a sixth part which was eight times the first [8], and a seventh part which was twenty-seven times the first [27]. After this he filled up the double intervals [i.e. between 1, 2, 4, 8] and the triple [i.e. between 1, 3, 9, 27] cutting off yet other portions from the mixture and placing them in the intervals, so that in each interval there were two kinds of means, the one exceeding and exceeded by equal parts of its extremes [as for example 1, 4/3, 2, in which the mean 4/3 is one-third of 1 more than 1, and one-third of 2 less than 2], the other being that kind of mean which exceeds and is exceeded by an equal number. Where there were intervals of 3/2 and of 4/3 and of 9/8, made by the connecting terms in the former intervals, he filled up all the intervals of 4/3 with the interval of 9/8, leaving a fraction over; and the interval which this fraction expressed was in the ratio of 256 to 243. And thus the whole mixture out of which he cut these portions was all exhausted by him. This entire compound he divided lengthways into two parts, which he joined to one another at the centre like the letter X, and bent them into a circular form, connecting them with themselves and each other at the point opposite to their original meeting-point; and, comprehending them in a uniform revolution upon the same axis, he made the one the outer and the other the inner circle. Now the motion of the outer circle he called the motion of the same, and the motion of the inner circle the motion of the other or diverse. The motion of the same he carried round by the side to the right, and the motion of the diverse diagonally to the left. And he gave dominion to the motion of the same and like, for that he left single and undivided; but the inner motion he divided in six places and made seven unequal circles having their intervals in ratios of two and three, three of each, and bade the orbits proceed in a direction opposite to one another; and three [Sun, Mercury, Venus] he made to move with equal swiftness, and the remaining four [Moon, Saturn, Mars, Jupiter] to move with unequal swiftness to the three and to one another, but in due proportion.

We are interested in the five initial concepts:

```
same, other, essence, form and nature
```

We also want to keep in mind
that Timaeus's creation of the world
is narrated around numbers.
And:
these numbers are of a quite different kind
to our understanding of numbers today.
Greek numbers are not iterative

and they are not starting with a 0:

```
0, 1, 2, 3, 4 …
```

They start with a part of the whole
and are working with magnitudes of 2 and 3:

```
2, 3, 4, 9, 8, 27 …
```

which is

```
2, 3, 2*2, 3*3, 2*2*2, 3*3*3 …
```

which the Greeks call
the double and the triple intervals.

We would say
these multiplicities of the same
are self-references of different orders.

Therefore it is of some importance
not to think about Greek numbers
as an interplay of ciphers (0 … 9),
but as an interplay of two principal characters:

```
2 and 3
```

These two characters
are complemented by the

```
1
```

and as a triple

```
2
3
1
```

they can be characterized as

```
same
other
essence
```

There are also three principal operations
on these characters:

```
multiplication
division
equivalence
```


which again are characterized as
the same, the other, and the essence.

To help us further
understand how to mingle
the character-numbers,
the Timaeus *only gives a few hints.*
A more explicit description
of the same stage play
within the Greek body of thinking
can be found
in the Pythagorean harmonic order.

This is the Pythagorean stage play,
or this is

```
how
the other (3)
looks at
the same (2)
in their multitudes
```

The magnitude between
the first multitudes of 3 and 2
is written as:

```
3/2
```

The magnitude between
the second multitude of 3 and 2:

```
9/4
```

The magnitude between
the third multitude of 3 and 2:

```
27/8
…
```

That is not enough.
There is another actor,
the essence,
the part of the whole,
the

```
1
```

And this is the stage play
of these three actors:

```
how does
the essence (1)
look at
the other (3)
look at
the same (2)
in their multitudes
```

The magnitude between (the magnitude between the first multitudes of 3 and 2) and (the magnitude between the part of the whole and the part of the whole)

```
(3/2)/(1/1) = 3/2
```

The magnitude between (the magnitude between the second multitudes of 3 and 2) and (the magnitude between the first multitudes of 2 and the part of the whole)

```
(9/4)/(2/1) = 9/8
```

The magnitude between (the magnitude between the third multitudes of 3 and 2) and (the magnitude between the first multitudes of 2 and the part of the whole)

```
(27/8)/(2/1) = 27/16
(81/16)/(4/1) = 81/64
(243/32)/(4/1) = 243/128
(729/64)/(8/1) = 728/512
```

And of course also the same (2) is looking at the other (3) and perceives other magnitudes.

```
how does
the essence (1)
look at
the same (2)
look at
the other (3)
in their multitudes
```

The magnitude between (the magnitude between the first multitudes of 2 and 3) and (the magnitude between the part of the whole and the first multitude of the 2)

```
(2/3)/(1/2) = 4/3
```

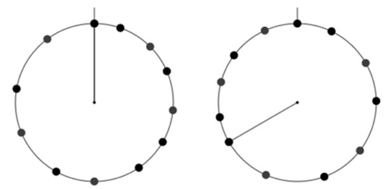

The magnitude between (the magnitude between the second multitudes of 2 and 3) and (the magnitude between the part of the whole and the second multitude of the 2)

```
(4/9)/(1/4) = 16/9
```

The magnitude between (the magnitude between the third multitudes of 2 and 3) and (the magnitude between the part of the whole and the second multitude of the 2)

```
(8/27)/(1/4) = 32/27
(16/81)/(1/8) = 128/81
(32/243)/(1/8) = 256/243
(64/729)/(1/16) = 1024/729
```

If one puts these ratios (multitudes) into a circle,
one gets the well-known contemporary illustrations
of the harmonic order,
of these two series of magnitudes
circling the interval between 1 and 2.



Of course we do not claim
that this is the only possible reading
of the *Timaeus*.
Rather, we challenge
this masterpiece of Western thinking
in a way that seems interesting to us.
And we hope that
staging this play in this way
would be interesting for Plato as well.

With this understanding
we again read the beginning of the *Timaeus*
to get an idea of the interplay
of the five concepts
same, other, essence, form, and nature.

He took the three elements of the `same`*, the* `other`*, and the* `essence`*, and mingled them into one* `form`*, compressing by force the reluctant and unsociable* `nature` *of the other into the same.*

As an example we take this equation:

```
(16/81)/(1/8) = 128/81
```

We have the five concepts:

The **`same,`**
the multitudes,
can be seen as

```
16 = 2*2*2*2
81 = 3*3*3*3
```

or as the principal character

```
2
```

The **`other,`**
the magnitude,
can be seen as
the ratio between the multitudes

```
16/81
```

or as the principal character

```
3
```

The **`essence,`**
the principle ratio,
can be seen
toward the part of the whole:

```
1/8
```

or as the characteristic,
or the modul,

```
1
```

The **`form`**
can be seen
as the result:


```
128/81
```

And finally the **`nature,`**
the incorporated arithmetics,
can be seen
as the way of articulating,
of shaping the form:

```
(16/81)/(1/8)
```

Also, we do have:

2 as the same,
3 as the other,
1 as the essence,

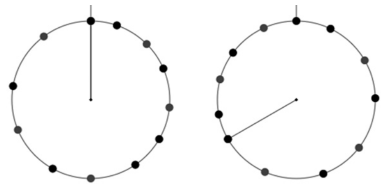
* as the multitude (same)
/ as the magnitude (other)
= as the essence:

Therefore the formula:

```
(16/81)/(1/8) = 128/81
```

can be read in this fugue:

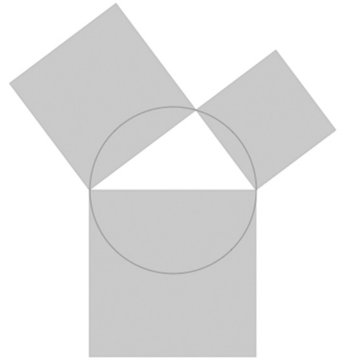
```
(((the multitudes of the same)
in magnitude to
(the multitudes of the other)
)
in magnitude to
(the essence
in magnitude to
(the multitudes of the same)
))
and
(((the multitudes of the other)
in magnitude to
(the multitudes of the same)
)
in magnitude to ((the multitudes of the same)
in magnitude to
the essence)
))
```



And finally
this might be
an adaptation of
our fugue
to the harmonic circle:

The `essence` might be the circle,
the `form` the rotation to a certain key,
and the `nature` as the pattern
that appears
as of points on the circle.

Therefore the
different characters,
the `same` and the `other`,
the 2 and 3,
are of the same essence,
but of different natures
(displayed as gray and black dots).
In music we know them
as major and minor.



We now want to use
the conceptual game above
to learn from the rationalization
of form in space that
Pythagoras established
with his famous theorem

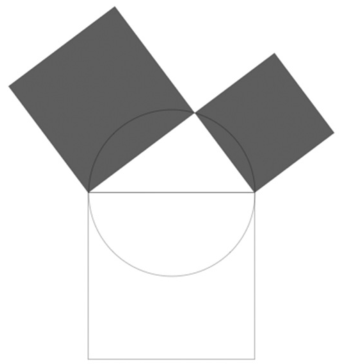
```
aa + bb == cc

or

3*3 + 4*4 == 5*5

or

3*3 + 2*2*2*2 == 5*5
```



This is our first obeservation:
a and b are of the *same*,
they are *multitudes*.
Whereas c is of the other,
a *magnitude*.

Or, if we want to stress
the concepts
of the same and the other further:

2, 3 and all their multitudes
are of a *finitude*,
whereas for example 5
as all the other primes
is not part of the finitude,
they are without parts,
they are of an *infinitude*.

A, B	C
multitude	magnitude
same	other
of the same	without parts
finitude	infinitude

This is the configuration
of more constitutional concepts
of our fugue:
In an *atomistic* setup
actors are of identical elements.
They are *identities*.
The *sensible* aspect of identities,
the *words*,
the *characters*,
or the *shapes*
of the actors,
take place on the *geometrical* stage.

The *intelligible* aspect of identities,
the *nature*,
the *essence*,
the *form*
of their phonetic talk,
take place on the *logical* stage.

Whereas in the inverse *axiomatic* setup
actors do not have parts,
they are indivisible,
they are *individuals*.

The *sensible* aspect of individuals,
the *forms*
of the *character's play*,
are orchestrated *arithmetically*.

The *intelligible* aspect of individuals,
the *shape*,
the *essence*,
of playing,
is orchestrated *algebraically*.

ATOMISTIC	AXIOMATIC
which is of the same	which has no parts
identities	individual
finite	infinite
characteristic forms	formal characters
natural shapes	shaped essence
sensible	intelligible
words	nature
characters	essence
geometry	logic
arithmetics	algebra
stage	orchestra

To complete our fugue:

With Pythagoras,
a master of an atomistic body of thinking,
the finite elements of the same
are understood as necessities,
as multitudes,
and from this thinking
the infinity of the one without parts
is looked at as a contingency,
as magnitude.

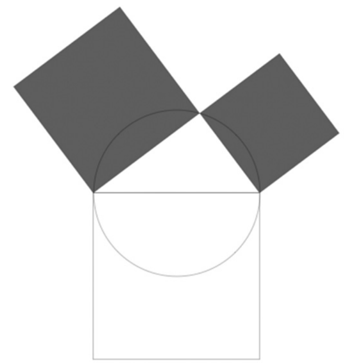
Therefore it is
within an atomistic body of thinking
that we say:
if an a and a b are of finite elements,
respectively multitudes,
and c is of an infinity,
respectively a magnitude.

Anticipating the arguments of the following text,
we find an inverse stage play
with Ptolemy,
a master of an axiomatic body of thinking.

The infinity of the one without parts
is looked at as necessity,
as multitude,

and from this thinking
the finite elements of the same
are looked at as contingencies,
as magnitudes.

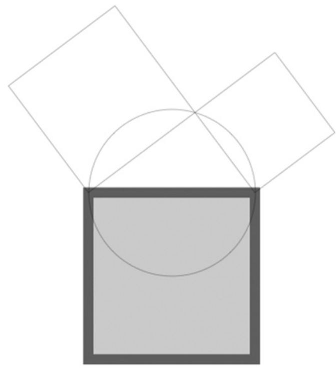
We now complete
the composition
of our fugue in detail.



a and b,
the multitudes,
act on the geometrical stage,
the finitude,
as identities,
as names,
in the shape
of filled squares.

a and b,
the multitudes,
play within the arithmetical orchestra,
the infinitude,
as an identity,
as numbers,
as a multiplicity
of the principal characters
2 and 3.

SENSIBLE OF THE MULTITUDE	
geometrical stage	arithmetical orchestration
finitude	infinitude
names	numbers
shape	characters
filled squares	multitudes of 2, 3



c,
the magnitude,
acts on the geometrical stage
as an individual
in the *form*
of an outlined square
between the shapes
of the two identities/multitudes.

Known elements to count on,
identities,
have shapes,
whereas unknown
elements to be measured,
individuals,
have forms.

Geometry measures
the endless space
between identities
within the infinite.

Geometry uses
logic on identities
to rationalize
the forms
of space
on the atomistic stage.

Within the arithmetical orchestration
c is articulated
by a formula or algorithm

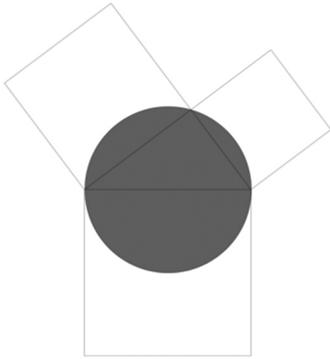
```
2 2 2 2 + 3 3 == 5 5
```

which is between the characters
of the two identities/multitudes.

Known elements to count on,
identities,
have characters,
whereas unknown
elements to be measured,
individuals,
have formulas.

Arithmetics measures
the endless space
between identities
within the infinite.
Arithmetics uses
algebra on identities
to rationalize
the formulas
of space
on the axiomatic stage.

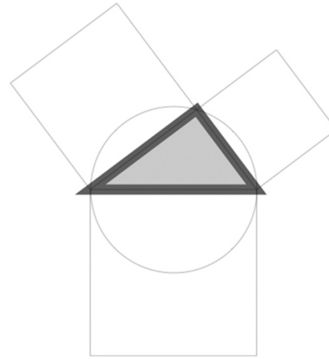
C	
SENSIBLE MAGNITUDE	
geometrical stage	arithmetical orchestration
form	formula
outlined square	2 2 2 2 + 3 3 == 5 5



Staging a and b
as intelligible multitudes,
which we call identities,
we are looking for
something like
the shape of logic,
or the shape of nature.
We suggest
to mask it with
a filled circle.

Orchestrating a and b
as an intelligible identity
we are looking for
something like
the *character of algebra*
or the *character of the essence.*
This should be
the essence of all multitudes,
the 1,
the module.

A, B	
INTELLIGIBLE MULTITUDE	
logical stage	algebraic orchestration
shape of logic	character of algebra
shape of nature	character of essence
filled circle	1



To stage c
as the intelligible magnitude,
as an identity,
which would be something like
the *form of logic*,
or the *form of nature*,
with Pythagoras
we can find the ratio
between the multitudes
by rational cuts of a circle,
or an outlined triangle.

Orchestrating c
as an intelligible multitude,
as an individual,
which would be something like
the *formula of algebra*,
the *formula of the essence*,
we gain the equivalence relation.

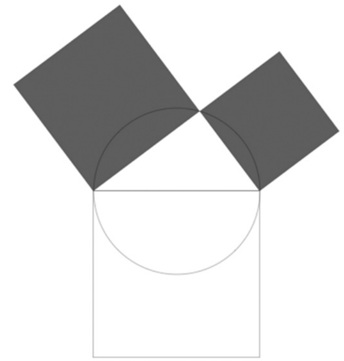
C	
INTELLIGIBLE MAGNITUDE	
geometrical stage	algebraic orchestration
form of logic	formula of algebra
form of nature	formula of essence
outlined triangle	==

PYTHAGORAS			
SENSIBLE			
A, B		C	
multitude		magnitude	
geometry	arithmetics	geometry	arithmetics
stage	orchestration	stage	orchestration
shape	characters	form	formula
filled squares	2, 3	outlined square	2 2 2 2 + 3 3 == 5 5

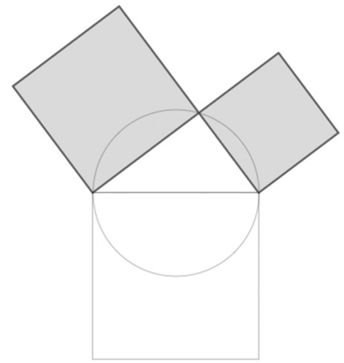
PYTHAGORAS			
INTELLIGIBLE			
A, B		C	
multitude		magnitude	
logic	algebra	logic	algebra
stage	orchestration	stage	orchestration
shape of nature	essential character, module	form of nature	essence of formula, equality
filled circle	1	outlined triangle	==

Thus far these assignations,
understood as
the first voice
of the composition
of our fugue.

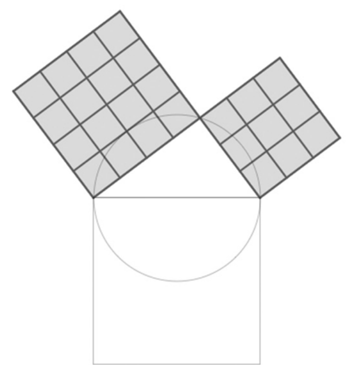
The multitudes a and b
can be seen



either as the geometrical shape of the same
(filled square),



as the geometrical form of the other
(outlined square)

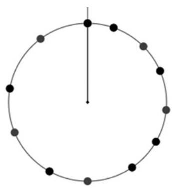


or as the essence,
a multitude of modules
(a rationalized array
of logical shapes, i.e. filled circles),
which we would like to name
ideal shape.

These simultaneous levels of abstraction
are of major importance for this text,

they are the key
to synchronizing
the different voices
of our fugue.

To close the circle
with the harmonic order
of Pythagoras.



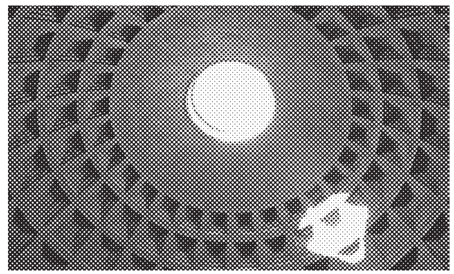
If the circle
is the logical form
to sense nature—
or we want to say:
it is the cipher of nature—

then the harmonic circle,
in its different rotations,
provides rational keys
or the characters
to realize the form of nature,
or: to render the logical to geometrical form.

Therefore:
The 1
is the key

to characterize the *universe*,
the 2 and the 3
are the elements
to encrypt the *world.*

With the
Greek temples
we find an architectonic articulation
where the sensible is primary:
where characteristic,
modulated geometrical shapes
are staging the geometrical form
of an arithmetical formula.

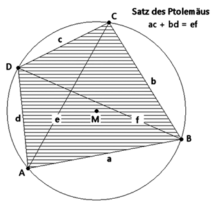


Whereas with the Roman Pantheon,
several hundred years later,
the intelligible becomes primary:
a modular, characterized logical shape,
the circle,
is orchestrating logical forms
around an algebraic equality,
a centered void.

III PTOLEMY

600, 800 years later
we find an inverse world.

We will choose the theorem
of Ptolemy (c. 90 CE–c. 168 CE)
to discuss this inversion.



Like the theorem of Pythagoras,
this theorem is working
with triangles and circles,
with the same, the other, and the essence.

But: unlike Pythagoras
Ptolemy does not rely
on the characteristic
or modularized
shape of things
(filled squares, filled circle)
to rationalize the form in between
(outlined squares, outlined triangle)
to generate identities,
which are of the same.

Ptolemy relies on
the rationalistic
or equalized forms
(the outlined triangles, outlined circle)
to analyze the shape within
(filled triangles, filled square)
to specify individuals,
which have no parts.

PYTHAGORAS	PTOLEMY
500 BCE	100 CE
analytic shape	rationalistic form
rationalize form	analyze shape
in between	within
generate	specify
identities	individuals

The casts of multitude and magnitude
have swapped their roles completely.

In today's notation
Ptolemy's equation is

```
ac + bd == ef
```

Pythagoras used the multiplicity
of the two characteristic elements

```
2 and 3
```

as his necessities.
the "tools" he relies on,
to measure the in-between

```
5
```

whereas Ptolemy's anchor points
are two by two of these equations, each of which had
been the form,
algorithm, and essence of Pythagoras:

```
aa + bb == ee
and
cc + dd == ee
and
aa + dd == ff
and
cc + bb == ff

They are intermingled
```

toward the one formular

```
ac + bd == ef
```

The equality of Pythagoras,
the magnitude of the essence,
and the 1 of Pythagoras,
the multitude of the essence,
are the points of inversion
from Pythagoras toward Ptolemy.

The same and being of the same,
the identity of Pythagoras,
is inverted into
the one that has no parts:
the individual of Ptolemy.

Now the multitudes
are no longer modularized characters

```
2,3
```

they are modulated formulas,

```
ad + be == cf
```

and the magnitudes
are no longer rationalized forms
of distinction of the identical

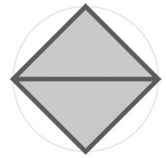
```
2 2  2 2 + 3 3 == 5 5
```

they are analyzed shapes
of equality of the individual,
which we know as the prime numbers
starting with 1

```
1, 2, 3, 5, 7, 11 …
```

PYTHAGORAS	PTOLEMY
modularized characters	modulated formulas
2, 3	ad + be == cf
rationalized forms of distinction	analyzed shapes of equality
2 2 2 2 + 3 3 == 5 5	1, 2, 3, 5, 7, 11 …
identical	individual

*This is the composition
of the second voice
of our fugue in detail.*



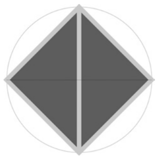
With this interchange of casts
the Ptolemy scenario
is the inverted Pythagoras scenario:

Logic and algebra
now are on the side of the sensible:

The finitude of the multitude of the sensible
now is staged logically
in a form
of the known representation
as outlined triangles.

The infinitude of the multitude of the sensible
now is orchestrated
in an algebraic formula.

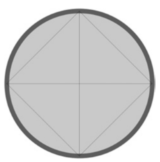
SENSIBLE MULTITUDE	
finitude	infinitude
logic	algebra
stage	orchestra
form	formula
outlined triangles	ab + cd == ef



The finitude of the magnitude of the sensible
now is staged logically
in the shape of filled triangles.

The infinitude of the magnitude of the sensible
now is orchestrated arithmetically
with modulations of the individuality,
the prime numbers.

SENSIBLE MAGNITUDE	
finitude	infinitude
logic	algebra
stage	orchestra
shape	modulation
filled triangles	1, 2, 3, 5, 7, 11…

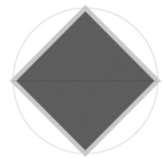


Geometry and arithmetics
are now on the side of the intelligible:

The finitude of the multitude of the intelligible
is staged geometrically
in the form
of an outlined circle.

The infinitude of the multitude of the intelligible
now is orchestrated
within the arithmetical balance.

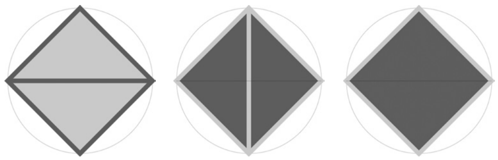
INTELLIGIBLE MULTITUDE	
finitude	infinitude
geometry	arithmetics
stage	orchestra
form	formula
outlined circle	balance (==)



The finitude of the magnitude of the intelligible
is staged geometrically
in the shape
of a filled rectangle.

The infinitude of the magnitude of the intelligible
now is orchestrated arithmetically
within the infinitesimal,
the generic.

INTELLIGIBLE MAGNITUDE	
finitude	infinitude
geometry	arithmetics
stage	orchestra
shape	modulation
filled rectangle	generic (∞)



Therefore the multitudes

```
aa + cc == ee

and

bb + dd == ff
```

can be seen

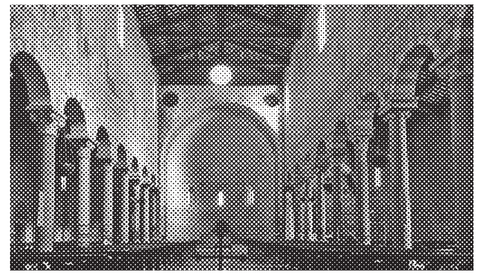
either as the logical forms
of the same
(outlined triangles),

as the logical shape
of the other within the same
(filled triangle),

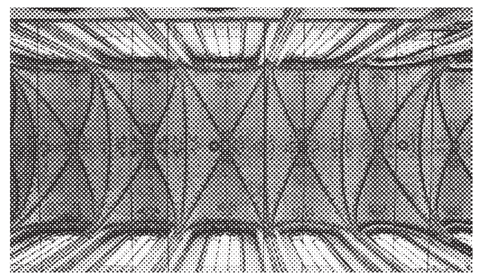
or as the essence,
a multitude of modules
(a rationalized array
of geometrical forms, i.e. outlined circles),
which we would like to name
ideal form.

same	other	essence
	PYTHAGORAS	
geometrical shape	geometrical form	ideal shape
filled square	outlined square	array
	PTOLEMY	
logical form	logical shape	ideal form
outlined triangles	filled triangles	graph

There are two different plays
staged in Ptolemy's body of thinking,
depending on whether
the sensible or the intelligible
gets the primary role.



With the
Romanesque basilica
we find an architectonic articulation
where the sensible is primary,
where calculated, balanced logical forms
are staging the logical shape
of an algebraic mode.



Whereas with the Gothic cathedral,
several hundred years later,
the intelligible becomes primary:
a balanced, calculated geometrical form,
the circle,
is orchestrating geometrical shapes
around a generic arithmetics,
the infinite void horizon.

SENSIBLE			
multitude		magnitude	
stage	orchestration	stage	orchestration
PYTHAGORAS			
geometry	arithmetics	geometry	arithmetics
shape	characters	form	formula
filled squares	2, 3	outlined square	2 2 2 2 + 3 3 == 5 5
PTOLEMY			
logic	algebra	logic	algebra
form	calculus	shape	modus
outlined triangles	ab + cd == e f	filled triangles	1, 2, 3, 5, 7, 11 …

INTELLIGIBLE			
multitude		magnitude	
stage	orchestration	stage	orchestration
PYTHAGORAS			
logic	algebra	logic	algebra
shape	module	form	equality
filled circle	1	outlined triangle	==
PTOLEMY			
geometry	arithmetics	geometry	arithmetics
form	balance	shape	generic
outlined circle	==	filled lined square	∞

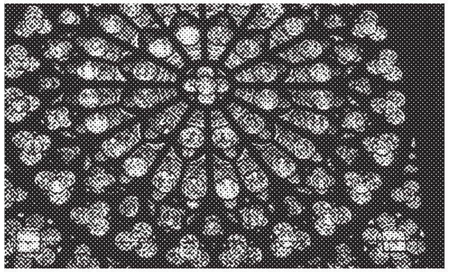
Pythagoras encrypts the universe with-out the 1
Ptolemy decrypts the cosmos from-in the ∞

Pythagoras is writing with an alphabet
of elementary characters (finitudes),
Ptolemy is reading the text
asking for axiomatic numbers (infinitudes).

Pythagoras is working
with the multitudes of 2 and 3,
Ptolemy is asking for the magnitudes of the primes:

1, 2, 3, 5, 7, 11 …
The ∞ is the text,
the cosmic characteristic,
the primes are the axioms
to decrypt the cosmos.

With Ptolemy
the outlined circle,
the void horizon,
is the ideal form
to sense nature,
to read the text of nature.
The different rotations
of this circle
are the rationalistic keys
to analyze the geometrical shape of nature:
filled lined squares.
Whereas the Roman Pantheon
brings the characterization
of the logical shape
to an infinite

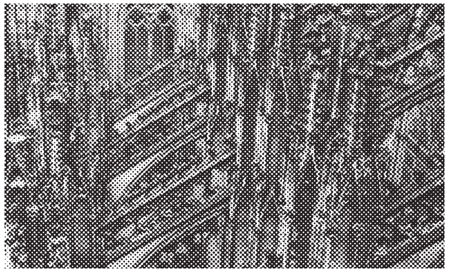


and articulates
a centered void
within the filled circle
as a new, a logical form
that we presented as
the outlined triangle,

the Gothic cathedral
brings the analysis
of the geometrical form
to an infinite
and orchestrates
a line around the void-circled horizon

(e.g. the Gothic rosette window)
as a new geometrical shape

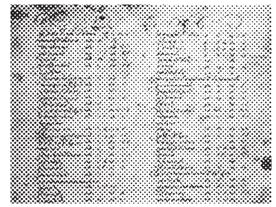
which we presented as
the filled lined square
(the Gothic tracery and buttress).



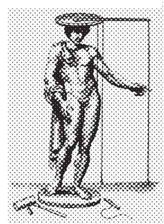
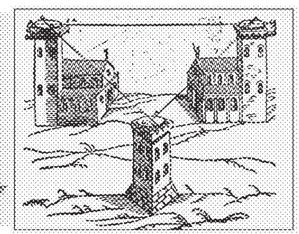
IV ALBERTI

Centuries later.
The Italian humanist Leon Battista Alberti
(1404–1472).

With him we see yet another inversion:
it is an inversion of Ptolemy
and a double inversion of Pythagoras.
To accomplish our fugue with another voice
we want to ask Alberti
and start with his measurement of the new Rome.

This is our voice of reference:
Ptolemy used an apparatus,
called dioptra,
to measure his position (magnitude)
within the stars (multitude).
And he created his famous map
as a list of pairs of two numbers
specifying the measured positions
of the important points
of his known world.

Alberti is using exactly the same apparatus,
but he is using it as an instrument:
he simply turns the dioptra
from the cosmic sphere,
the stars, and the primes,

to himself, moving,
or, to put it more simply,
to the ground.
In doing so he himself,
whose position was subject of measurement
with Ptolemy (= magnitude)
now becomes the point of stability
or the reference (= multitude)
to measure distances in between.

PTOLEMY	ALBERTI
apparatus	instrument
the stars	he himself
the position within	the distance in between

We want to describe
this inversion
more precisely.

Ptolemy uses an apparatus
to dissect his position
within the cosmic order
to construct
a map
of all positions
on a void plane.

An apparatus is:
on the sensible plane:
a logical form
of an algebraic calculus,
(
an outlined triangle:
the actual point
of measurement,
the calculus:
to get the position
within two triangles
)
on the intelligible plane:
a geometrical form
of an arithmetic balance

(
an outlined circle:
the disk
for any measurement,
the equality:
follow the same procedure
for each measurement
).

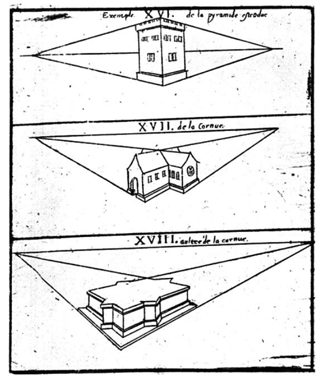
A map, an image, or a construction is:
on the sensible plane:
a logical shape
of an algebraic mode
(
filled triangles:
balanced figures
of the measured destinctions,
primes:
on fictional layers,
or species
)
on the intelligible plane:
a geometrical shape
of an arithmetical generation
(
filled square:
a distinctive shape.
infinite:
on void ground,
or:
a prediction
within the unknown,
or:
operating within modes/monas:
modulation
).

Alberti uses
Ptolemy's apparatus
as an instrument.

An instrument
contract distances
on worldly ground
to constitute
connections
around centered voids.

apparatus	instrument
dissect	contract
cosmic order	worldly ground
construction	constitution/model
position	connection
void plane	centered void

An instrument is:
on the sensible plane:
a geometrical shape
of arithemetical characters,
(
a filled square:
a distinct shape
on void ground.
however:
an assumption
instead of a prediction
)
on the intelligible plane:
a logical shape
of an algebraic module
(
a filled circle:
a generic figuration
for any contract,
== fugue
1:
follow the same procedure
for each measurement
== generic
).



De Artificiali Perspectiva, Pelerin (1505)

A model, a fugue, or a constitution is:

on the sensible plane:
a geometrical form
of an arithmetical formula
(
outlined square:
with Alberti, the lines,
in general, are a constellation,
the formula: in proportion
)

on the intelligible plane:
a logical form
of algebraic equality
(
outlined triangle:
with Alberti, the alignment,
in general, the adjustment,
the equality: in perspective.
Or:
acting with moduls,
modularization
).

map/image	model/icon
figure	fugue
specific figuration	proportional constellation
predictive distinction	perspective adjustment
operate	act
modulation	modularization

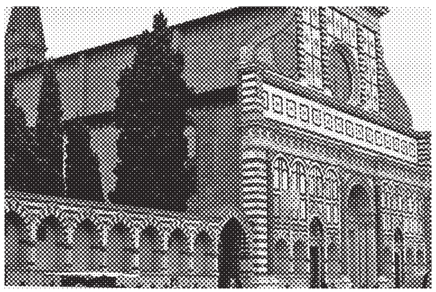
Alberti articulates
an inversion to Ptolemy
and an abstraction to Pythagoras.

The elements of Alberti's geometry
are coming out of
Ptolemy's balanced infinity,
the void horizon.

Alberti's geometrical stage
is the surface
of the balanced
filled volumes of Ptolemy.
It is the outline
of the geometrical shape of Ptolemy.

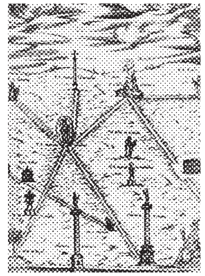
Alberti's stage
is in between the old cosmic order.

Alberti's basilica Santa Maria Novella
plays with
new lines
on the surface
of the old volumes.



With these lines
in between spaces
new Rome was built
in between the ruins
of antique Rome.
For that
Sixtus V, the gardener,
just planted a few obelisks
and lined up fresh water
at dedicated places.

The ruins and the spatiality of antique Rome
became a matter of spatial archaeological interest,
the new Rome the sensible lines of time in between.



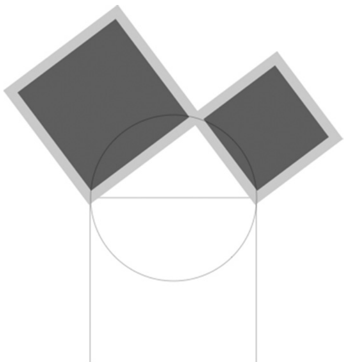
Alberti's elements
are spatial by nature
and Alberti is positioning them
on the geometrical stage
of time and movement.
This stage is symmetrical to
the geometrical stage of Pythagoras,
but:
The stage is of time
not of space.

The elements are of a spatial nature,
they are multitudes
of the primes,
the spatial algebraic modes.
They are not of a mythical nature
as they had been with Pythagoras
and his multitudes
of the 2 and the 3,
the mythical algebraic modes.

The elements are ciphers
around the 0,
not characters
around the 1.

PYTHAGORAS	ALBERTI
stage of space	stage of time
mythical elements	spatial elements
2, 3	a, b (primes)
around the 1	around the 0
character	cipher

The details
of the third voice
of our fugue:

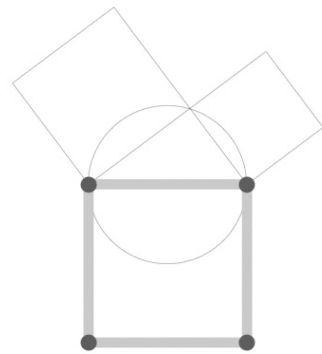


a and b,
the multitudes,
act on the geometrical stage,
the finitude,
as identities,
as names,
in the shape
of filled lines on squares.

a and b,
the multitudes,
play within the arithmetical orchestra,

the infinitude,
as an identity,
as numbers,
as a multiplicity
of the principal ciphers,
the primes
(we know this as infinite series).

SENSIBLE OF THE MULTITUDE	
geometrical stage	arithmetical orchestration
finitude	infinitude
shape	cipher
filled lines on squares	a, b (multitudes of primes)



c,
the magnitude,
acts on the geometrical stage
as an individual
in the *form*
of an outlined line on a square
between the shapes
of the two identities/multitudes.

Within the arithmetical orchestration
c is articulated
by a formula or *algorithm*

```
aa + bb == cc
```

which is between the ciphers
of the two identities/multitudes
(we know this as the proportion
of infinite series, e.g. Wallis 1656).

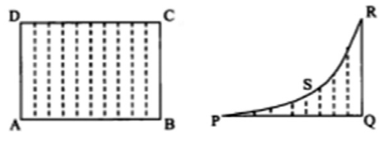
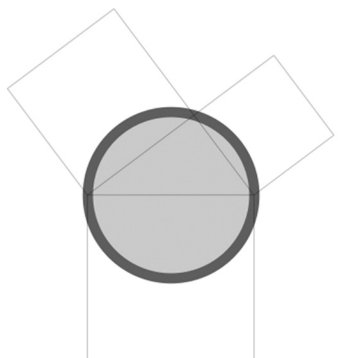


Fig. 3

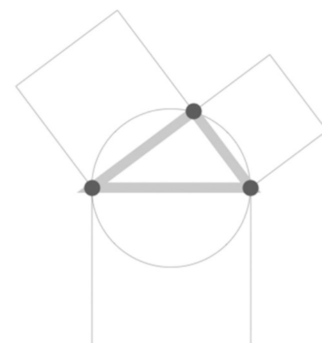
C	
SENSIBLE MAGNITUDE	
geometrical stage	arithmetical orchestration
form	formula
outlined line on a square	a a + b b == c c



Staging a and b
as intelligible multitudes,
which we call identities,
we are looking for
something like
the *shape of logic*,
or the *shape of nature*.
We suggest
the filled line on a circle.

Orchestrating a and b
as an intelligible identity
we are looking for
something like
the *character of algebra*
or the *character of the essence.*
This should be
the essence of all multitudes
the division by 1,
the 0,
the module.

A, B	
INTELLIGIBLE MULTITUDE	
logical stage	algebraic orchestration
shape of logic	character of algebra
shape of nature	character of essence
filled line on a circle	0

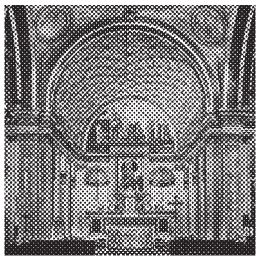


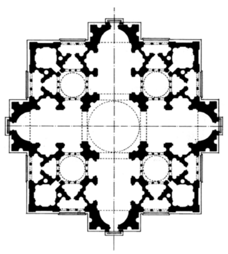
To stage c
as the intelligible magnitude,
as an identity,
which would be something like
the form of the logic,
or the form of nature,
with Alberti
we can find the ratio
between the multitudes

by rational cuts of
the lines on a circle,
or as points outlining a triangle.

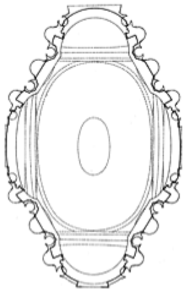
Orchestrating c
as an intelligible multitude,
as an individual,
which would be something like
the *formula of algebra*,
the *formula of the essence*,
we gain the equivalence relation.

c	
INTELLIGIBLE MAGNITUDE	
geometrical stage	algebraic orchestration
form of logic	formula of algebra
form of nature	formula of essence
points outlining a triangle	==





With Renaissance architecture
we find an architectonic articulation
on the stage of time
where the sensible is primary,
where characteristic,
modulated geometrical shapes
are staging the geometrical form
of an arithmetical formula.



Whereas
with Baroque architecture,
two hundred years later,
the intelligible becomes primary:
a modular, characterized, logical shape,
the circle,
is orchestrating logical forms
around an algebraic equality,
a centered void
in time.

V LAGRANGE

Again 300 years later:
With Lagrange's (1736–1813) interpolation
we position ourselves
in the inversion of Alberti,
a double inversion of Ptolemy,
and a triple inversion of Pythagoras.

Because of these symmetries
we can constitute this next voice of our fugue
with the help of the known equation

```
ac + bd == ef
```

Where

```
aa + bb == ee
and
cc + dd == ee
and
aa + dd == ff
and
cc + bb == ff
```

are the multitudes,
the same,
which have no parts.

This is our fugue for Lagrange
in line with Ptolemy:

As the instrument of Alberti
Lagrange's apparatus is working
with triangles and circles,
with the same, the other, and the essence.

But: unlike Alberti,
Lagrange does not rely on
the ciphered or modularized
shape of things
(filled lined squares, filled line circle)
to rationalize the form in between
(outlined line squares, outlined lined triangle)
to generate identities,
which are of the same.
Lagrange relies on

the rationalistic
or equalized forms
to analyze the shape within
to specify individuals,
which have no parts.

ALBERTI	LAGRANGE
15th century	18th century
analytic shape	rationalistic form
rationalize form	analyze shape
in between	within
identities	individuals

Even if it is a little cryptic,
to get to know liner algebra
we want to continue
to play the voice of our fugue.

The casts of multitude and magnitude
has interchanged their roles completely.

Lagrange's equation can be written as

```
ac + bd == ef
```

Alberti used the multiplicity
of the two series of primes

```
a and b
```

as his necessity,
the tools he relies on,
to measure the in-between

```
aa + bb == 1
```

with the rational number,
or the line in between,
as his cipher,

whereas Lagrange's anchor points are
multiple of these equations,
each of which had been the
form, algorithm, and essence
of Alberti:

```
aa + bb == ee
and
cc + dd == ee
and
aa + dd == ff
and
cc + bb == ff
```

They are intermingled
toward the one formula

```
ac + bd == ef
```

The equality of Alberti,
the magnitude of the essence,
and the 0 of Alberti,
the multitude of the essence,
are the points of inversion.
The same and being of the same,
the identity of Alberti,
is inverted by Lagrange into
the one that has no parts:
the individual.

Now the multitudes
are no longer modularized ciphers

```
a/b
```

(proportions of infinite series of primes),
they are modulated formulars,

```
ad + be == cf
```

and the magnitudes
are no longer rationalized forms
of distinction of the identical,

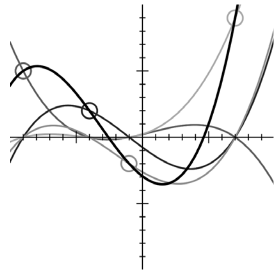
```
aa + bb == 1
```

they are analyzed shapes
of equality of the individual,
which we know as
the roots of the polynomials

```
ax + bx2 + cx3 …
```

starting with -1.

This is how Lagrange's interpolation
works in detail
to create a line (the black one),
which has multiple names,
which passes multiple points,
to orchestrate a set of points,
under the assumption of linearity.



We wish to interpolate $f(x) = x^3$ over the range $1 \leq x \leq 3$, given these three points:

$$x_0 = 1 \quad f(x_0) = 1$$

$$x_1 = 2 \quad f(x_1) = 8$$

$$x_2 = 3 \quad f(x_2) = 27$$

The interpolating polynomial is:

$$L(x) = 1 \cdot \frac{x-2}{1-2} \cdot \frac{x-3}{1-3} + 8 \cdot \frac{x-1}{2-1} \cdot \frac{x-3}{2-3} + 27 \cdot \frac{x-1}{3-1} \cdot \frac{x-2}{3-2}$$
$$= 6x^2 - 11x + 6.$$

$$\ell_j(x) := \prod_{\substack{0 \leq m \leq k \\ m \neq j}} \frac{x - x_m}{x_j - x_m} = \frac{(x-x_0)}{(x_j-x_0)} \cdots \frac{(x-x_{j-1})}{(x_j-x_{j-1})} \frac{(x-x_{j+1})}{(x_j-x_{j+1})} \cdots \frac{(x-x_k)}{(x_j-x_k)},$$

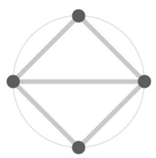
Lagrange fills the line
toward linearity
with lines in the sense of Alberti,
as Ptolemy filled the plane
with the ratios of Pythagoras.

By this method,
Lagrange does not get an identity
of a perspective line,
but an individual linearity,
called a dimension.
And he is able to do so
by specifying a formula
to transfer
one linearity
to another linearity
to establish

a movement without
movement in the sense of Alberti,
a fictional movement,
a movability
in time.

It is the story about
development and education in time:
to shape an individual
by feeding them with
more and more
points of truth
to decipher,
to analyze
the cosmic order.

ALBERTI	LAGRANGE
modularized ciphers	modulated formular
a / b	ad + be == cf
rationalized forms of distinction	analyzed shapes of equality
aa + bb == 1	ax + bx2 + cx3 ...
line	linearity
identical	individual



Logic and algebra
now are on the side of the multitude again,
and matter by necessity:

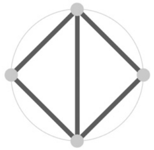
The finitude of the multitude of the sensible
now is staged logically
in a form
of the known representation
as points outlining a triangle.

(

Today we associate this
logical form
with a polyline
in the sense of
the polynomial interpolation of Newton.
)

The infinitude of the multitude of the sensible
now is orchestrated
in an algebraic formula,
the calculus.

SENSIBLE MULTITUDE	
finitude	infinitude
logic	algebra
stage	orchestra
form	formula
points outlining a triangle	AB + CD == EF



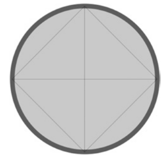
The finitude of the magnitude of the sensible
now is staged logically
in the shape of filled lines on triangles.

(

Today we associate
this logical shape
with the infinitesimal polynomial interpolation
in the sense of Leibniz,
if we think in contrast to Newton
)

The infinitude of the magnitude of the sensible
now is orchestrated algebraically
with modulations of the individuality,
the roots of the polynomial interpolation.

SENSIBLE MAGNITUDE	
finitude	infinitude
logic	algebra
stage	orchestra
shape	modulation
filled lines on a triangle	the polynomial ax + bx2 + cx3 …
polynomial interpolation	roots of the polynomial



The finitude of the multitude of the intelligible
is staged geometrically
in the form
of an outlined circle.

(

Today we would associate
this geometrical form
with the non-Euclidean geometry
of Carl Friedrich Gauss,

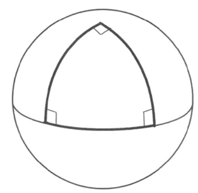
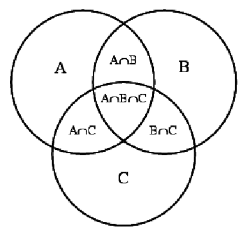
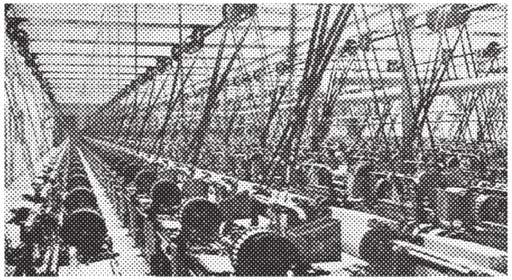


Figure 6.13 Triple-right triangle on a sphere

with the set theory,



with the capsulation
of energy and/or labor (self-movement),
we call a product,

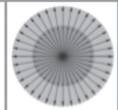
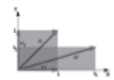


or with a pixel of a technical image
as an abstraction of Ptolemy's map.

)

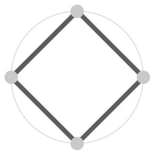
The infinitude of the multitude of the intelligible
now is orchestrated
within the arithmetical balance.

(
Today we would associate
this arithmetical balance
with the operations on matrices,
which are about orchestrating
coefficients of polynomials
in a dimensional order.

	scaling	uneaqual scaling
illustration		
matrix	$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$	$\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$
characteristic polynomial	$(\lambda_1 - k)^2$	$(\lambda_1 - k_1)(\lambda_1 - k_2)$
eigenvalues λ_i	$\lambda_1 = \lambda_2 = k$	$\lambda_1 = k_1$ $\lambda_2 = k_2$
algebraic multipl. $\mu_i = \mu\ (\lambda_i)$	$\mu_1 = 2$	$\mu_1 = 1$ $\mu_2 = 1$

)

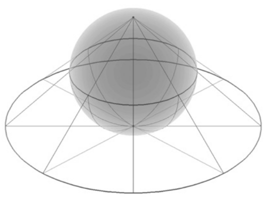
INTELLIGIBLE MULTITUDE	
finitude	infinitude
geometry	arithmetics
stage	orchestra
form	formula
outlined circle	balance (==)



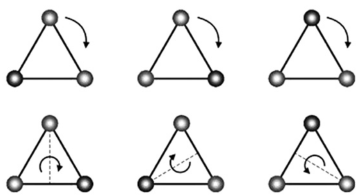
The finitude of the magnitude of the intelligible
is staged geometrically
in the shape
of filled lines on a rectangle.

(

Today we would associate
the algebraic equality
with Riemann Geometry,

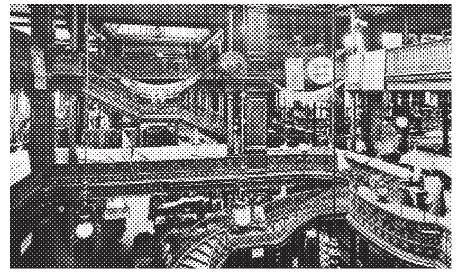


with the Group Theory of Galois,



with the brands and labeling of products,

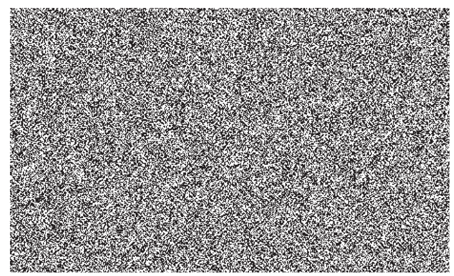
and with the malls
as the complementary part
to factories.



Le Bon Marché
)

The infinitude of the magnitude of the intelligible
now is orchestrated arithmetically
within the infinitesimal,
the generic.

(
This can be associated
with the concept of entropy,
a fully equally distributed state.



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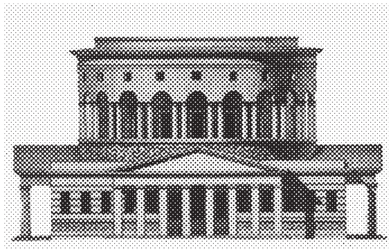
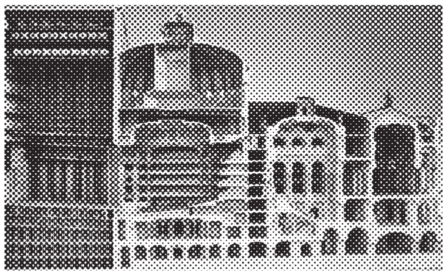


With Ledoux’s Rotonde de la Villette
we find an architectonic articulation
where the sensible is primary
and where calculated,
balanced logical forms are staging the logical shape
of an algebraic mode,

Palais Garnier



whereas with Le Bon Marché,
or the Palais Garnier, Paris,
one hundred years later,
the intelligible becomes primary:
a balanced, calculated geometrical form,
the circle,
is orchestrating geometrical shapes
around a generic arithmetics,
the infinite void horizon.

Alberti encrypts the universe *with-out* the o,
Lagrange decrypts the cosmos *from-inside* the ∞.

Alberti is writing with an alphabet
of elementary ciphers (finitudes),
Lagrange is reading the text
asking for the axiomatic roots (infinitudes).
Alberti is working
with the multitudes of the primes,
Lagrange is asking
for the magnitudes of the polynomial roots.

The ∞ is the text,
the cosmic encryption;
the polynomial roots are the axioms
to decrypt the cosmos.
With Lagrange
the outlined circle,
the void horizon,
is the ideal form
to sense nature,
to read the text of nature.

The different rotations,
of this circle
are the rationalistic keys
to analyze the geometrical shape of nature.
Filled pointed squares.

Whereas the Roman Pantheon
brings the characterization
of the logical shape
to an infinite
and articulates
a centered void
within the filled circle
as a new, a logical form,
which we presented as
the outlined triangle;

whereas the Gothic cathedral
brings the analysis
of the geometrical form

to an infinite
and orchestrates
a line around the void-circled horizon
as a new geometrical shape,
which we presented as
the filled lined square;
whereas Baroque architecture
brings the characterization
of the logical shape
to an infinite
and articulates
a centered void
within the filled circle
as a new, a logical form,
which we presented as
points outlining a triangle;

the opera house,
or the factory hall,
or the exhibition hall,
brings the analysis
of the geometrical form
to an infinite
and orchestrates
a line around the void-circled horizon
as a new geometrical shape,
which we presented as
the filled pointed square.

VI MARKOV

Finally, we turn
toward information
and the quantum.

Why Andrej Andreyevich Markov?
We could follow Wiener,
Turing, or Shannon?
Even Chomsky?
Because we asume
they think
of computers as machines
running positively
on an entropic background.
They adhere to the idea
of a computer,
of information
as being meaningful in time.

"Information is neither matter nor energy."
—*Norbert Wiener*
Cybernetics: Or Control and Communication
in the Animal and the Machine, 1948.

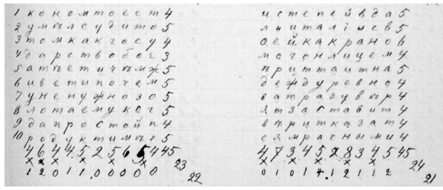
With our fugue
we expect
an expulsion
from entropy.
As Alberti is expelled from
Ptolemy's spatial cosmic order,
we expect to be expelled from
Lagrange's chronological analytical order.

We do not expect to reflect on entropy,
we are looking for entropic projections,
as we find them in quantum physics,

Richard Feynman's
Strange Theory of Light and Matter, 1985,

or cryptography.

Or with Markov
and the operational principals
of social media.



Alexandr A. Markov's stochastical analysis of the epos "Evgenij Onegin" of Alexander Sergeyevich Pushkin, 1913.

Markov simply cuts
the famous epos by Alexander Pushkin
into meaningless consonants and vowels,
counts the characters,
analyses the numbers,
and gets values of probabilities,
by which one can navigate the text
in a stable and ordered way
prior to any specificity,
prior to any reading
or understanding.

This is the birth
of a new geometry
beyond time.
And this is how
Google's PageRank
and social media
work today.
Unlike Wiener, Neumann,
Turing, Shannon, or Chomsky,

Markov,
like Dedekind or Riemann,
is not embedded within entropy.
Markov simply cuts entropy
and keeps the parts,
as they are:
entropies.
But he gains the cuts
and he is able to work with them
in a meaningful way.

Alberti took the cosmic reflective series,
the rationality of Ptolemy.
On the sensible plane
he anchored it
as geometry and logics
as multiples
to the ground.
On the intelligible plane
he aligned it
as arithmetics and algebra
with the infinite horizon.

Therefore by modernity
the entity of a rational number,
a 2/3,
which consists of two natural numbers
(characters, not numbers),
literally cuts the Ptolemean cosmos
of series of primes
into two
and puts them into proportion.
The world of character determination,
the infinity of spatial order,
is cut into two,
the parts are ciphered by numbers,
and arranged in time.
This constitutes a modernity in time.

And this is
how Markov, information, and the quantum
sound:

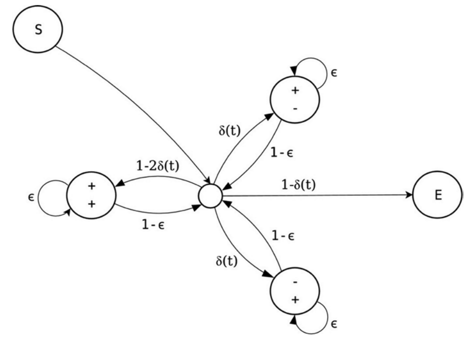
Markov took the entropic analytical functions,
the rationality of Lagrange.
On the sensible plane
he anchored it
as geometry and logics
as multiples
to the ground.
On the intelligible plane
he aligned it
as arithmetics and algebra
with the infinite horizon.

Therefore with the digital,
the entity of a signature
like 1, 0, 0, 1,
which is a proportion
of two numbered species,
literally cuts the analytical cosmos
of entropic functions
into two
and puts them into proportion.
The world of numeric specification,
the infinity of the chronological order
is cut into two,
the parts are subscribed
and arranged in,
as we suggest,
probability values.
This constitutes modernity in value.

ALBERTI	MARKOV
cipher	signature
series of primes	entropy of functions
2/3	0,1
modernity in time	modernity in value

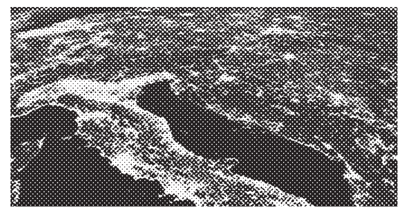
The symmetries
of the rational triangles in space
of Pythagoras,
the perspective triangles in time
of Alberti
and the probabilistic triangles in value
of Markov chains
are striking.

In the typical diagrams
of the Markov chain
we see
the geometrical multitudes
of analytical elements (peripheral circles)
and the magnitude
of a digital element in between (centered circle),
and we have the arthmetics of probabilites,
as a glue,
as the magnitude
in between the multitudes.



This is how we
can read the
Internet, mobiles, social media:
analytical, energized elements,
connected by necessities (multitudes)
on the electrical level,
and mediatized
and operated by contingencies (magnitudes),
and glued to the world of all the other nodes
by probabilities.

The any moves within the every.
Anybody googles everybody.
A new identiy is created upon every individuality.

What is information then?

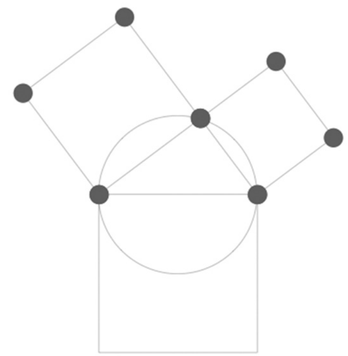
With Pythagoras we had
a geometry in between things,
with Alberti
a geometry in between spaces,
with Markov
a geometry in between times.

Information
is a geometry
in between times.

But how to operate
on information,
if it is in between times,
if information is
neither matter
nor energy,
if computers are
not machines?

If we look at Markov as a protagonist
of the multitude of the intelligible of information,
we suggest Kohonen
and his Self-Organizing Maps
as a protagonist
of the magnitude of the intelligible of information.

Teuvo Kohonen, Self-Organisation and Associative Memory, Springer, Berlin, 1983



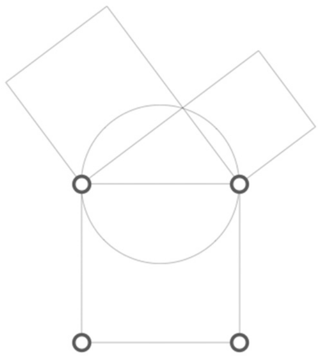
A and B,
the multitudes,
act on the geometrical stage,

The finitude,
as identities
in the shape
of filled points on squares.
(
We know this as
a group,
a dimension,
a technical image,
a technical infrastructure.
)

A and B,
the multitudes,
play within the arithmetical orchestra,
the infinitude,
as an identity,
as a multiplicity
of the principal signatures
the polynominal roots.

(
We know this as
energized
and optimized
elements.
)

SENSIBLE OF THE MULTITUDE	
geometrical stage	arithmetical orchestration
finitude	infinitude
shape	signature
filled points on squares	A, B (multitudes of optimized elements)

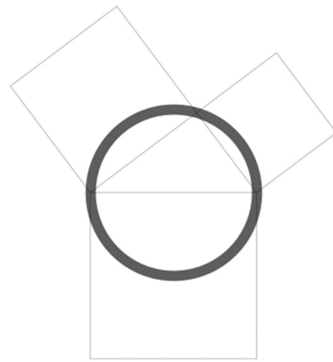


C,
the magnitude,
acts on the geometrical stage
as an individual
in the form
of outlined points on a square
between the shapes
of the two identities/multitudes.
(
We know it
e.g. as wavelets.
)

Within the arithmetical orchestration
C is articulated
by a formula or *algorithm*
AA + BB == CC
which is between the signatures
of the two identities/multitudes.

(
We know it as
categories
)

C	
SENSIBLE MAGNITUDE	
geometrical stage	arithmetical orchestra
form	formula
outlined points on a square	AA + BB == CC

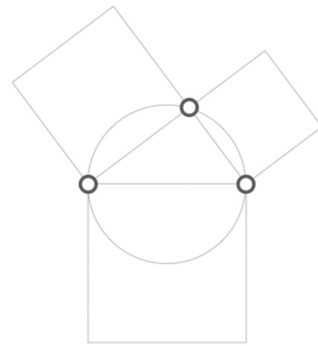


Staging A and B
as intelligible multitudes,
which we call identities,
we are looking for
something like
the *shape of logic*,
or the *shape of nature*.
We suggest
the filled points on a circle.
(
A circle of probabilities
as we know it from Google,
providing the probabilities
toward the whole world
to any statement.

We know this as
Markov chains.
)

Orchestrating A and B
as an intelligible identity,
we are looking for
something like
the *character of algebra*
or *the character of the essence.*
This should be
the essence of all multitudes,
the division by zero,
a 0/0,
the digital module.
(
We divide
any statement
by the index
to any element
of the world.
)

A, B	
INTELLIGIBLE MULTITUDE	
logical stage	algebraic orchestration
shape of logic	character of algebra
shape of nature	character of essence
filled points on a circle	0 / 0



To stage C
as the intelligible magnitude,
as an identity,
which would be something like
the *form of the logic*,
or the *form of nature*,
with Markov
we can find this with the ratio
between the multitudes
by rational cuts of
the points on a circle,
or as outlined points on a triangle.

(
This is, how we would discuss
Kohonen's self-organizing maps
)

Orchestrating C
as an intelligible multitude,
as an individual,
which would be something like
the *formula of algebra*,
the *formula of the essence*,
we gain the equivalence relation.

(
The sum of
the probabilities
of the whole world
to any statement
keeps the one.
)

C	
INTELLIGIBLE MAGNITUDE	
geometrical stage	algebraic orchestration
form of logic	formula of algebra
form of nature	formula of essence
outlined points on a triangle	==

If the Euclidean model
articulates
the logical form of mythical elements in space

and

if the perspective model
articulates
the logical form of spatial elements in time

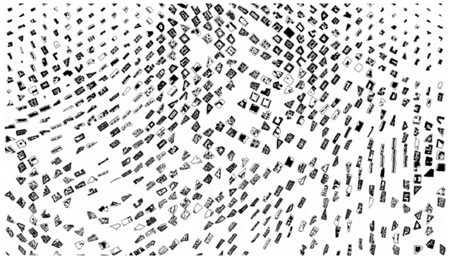
then

the self-organizing map
articulates
the logical form
of chronological elements
in probability values.

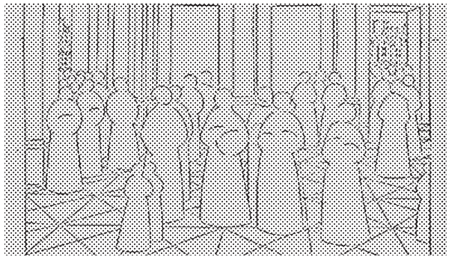
Therefore we suggest
that we should not
talk about
a self-organizing map
but a *self-organizing model.*

An analytical map of Zurich,
which is stable
in the analytical/chronological order.

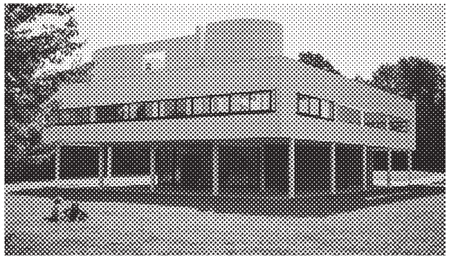
And in inversion to the map,
a self-organized model of Zurich,
which changes the constallation of elements
according to the
analytical/chronological position
of the observer.



Exactly symmetrical to
the Renaissance model in time,
which changes the constellation of elements
according to the
spatial position
of the observer.



De Artificiali Perspectiva, Pelerin (1505)



With the
architecture of the twentieth century
we find an architectonic articulation
on the stage of probability values,
where the sensible is primary,
where the optimized,
modulated geometrical shapes
are staging the geometrical form
of an arithmetical formula
around probabilities.

This view toward architecture
is in sync
with the architecture of the Renaissance
and with the architecture of the ancient Greeks.
but on different levels of abstraction.
What we are expecting for twenty-first-century
architecture
is that the intelligible becomes primary:
a modular, characterized, logical shape,
the circle,
is orchestrating logical forms
around an algebraic equality,
a centered void
in value.

We are expecting a move
toward a falling in sync
with the architecture of the Baroque
and with the architecture of ancient Rome,
but on different levels of abstraction.

If we are right with our fugue,
then the primacy of
the *sensible in the architecture*
of the twentieth century,

the geometrical shapes,
(
we know them as wavelets
and in an applied form:
as parameters,
(referring to Alberti's geometry
as the multiplicities of the code)
or grammars (incl. L-systems, GA, CA …),
(referring to Lagrange's arithmetics
as the multiplicities of the code)
)
will shift toward a primacy
of the *intelligible in architecture*,
toward logical forms,
toward a digital Baroque,
in the twenty-first century.

Self-Organizing Models,
in implementations
like Kohonen's maps,
will be the active subjects,
contracting natures,
to explore
the new world
beyond time.

SENSIBLE				INTELLIGIBLE			
MULTITUDE		MAGNITUDE		MULTITUDE		MAGNITUDE	
stage	orchestration	stage	orchestration	stage	orchestration	stage	orchestration
PYTHAGORAS							
geometry	arithmetics	geometry	arithmetics	logic	algebra	logic	algebra
shape	characters	form	formula	shape	module	form	equality
filled squares	2, 3	outlined square	2 2 2 2 + 3 3 == 5 5	filled circle	1	outlined triangle	==
PTOLEMY							
logic	algebra	logic	algebra	geometry	arithmetics	geometry	arithmetics
form	calculus	shape	modus	form	balance	shape	generic
outlined triangles	ab + cd == ef	filled triangles	1, 2, 3, 5, 7, 11 ...	outlined circle	==	filled lines on squares	∞
ALBERTI							
geomety	arithmetics	geomety	arithmetics	logic	algebra	logic	algebra
shape	ciphers	form	formula	shape	module	form	equality
filled lines on squares	a,b	outlined lines on squares	aa + bb == cc	filled line on a circle	0	points outlining a triangle	==
LAGRANGE							
logic	algebra	logic	algebra	geometry	arithmetics	geometry	arithmetics
form	calculus	shape	modus	form	balance	shape	generic
points outlining a triangle	AB + CD == EF	filled lines on triangles	the roots of polynomes, ax + bx2 + cx3 ...	points outlining a circle	==	filled pointed square	∞
MARKOV							
geometry	arithmetics	geometry	arithmetics	logic	algebra	logic	algebra
shape	signature	form	formula	shape	module	form	equality
filled points on squares	A,B	outlined points on squares	AA + BB == CC	filled points on a circle	0,0	points outlining a point	==

Characters in space
Euclidean Geometry

Spatial ciphers in time
Analytical Geometry

Chronological signatures in value
Operational Geometry (Code)